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Entangling Mirrors

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Motivation

*) Entanglement has been investigated extensively in many body systems.

*) We have witnesses and measures of entanglement as well as some (but <u>not</u> much) experimental evidence

*) Want to investigate quantum → classical transition.

Heuristics

Expect Q to C transition

when:

$$N, S, T, m, d \to \infty$$

$$t \to \infty, \quad \hbar \to 0$$

For example: looking at BEC, entanglement appears when.

$$T < T_c = \frac{\hbar^2 \rho^{\frac{2}{d}}}{2m k_B}$$

$T_c \to 0$ when $m \to \infty$

$\hbar \to 0$

$d \to \infty$

Anders et al, N. J. Phys. (2006).

Decoherence

$$\frac{t_{dec}}{t_{diss}} = \frac{\hbar^2}{2m(\Delta x)^2} \frac{1}{KT}$$

ex. $m = 10^{-12}\,Kg$

$\Delta x = 1\,mm$ $\quad T = 1K$

$$\frac{t_{dec}}{t_{diss}} = \frac{10^{-68}}{10^{-12}\,10^{-6}}\,10^{23} = 10^{-27}!$$

so, if $t_{diss} = 10^{-8}s \Rightarrow \underline{\underline{t_{dec} \sim 10^{-19}s!}}$

Amico, Fazio, Osterloh, Vedral, Rev. Mod. Phys. (2008).

Problem

Can photon pressure create entanglement between a (macroscopic) mirror and light field?

Can it subsequently be used to entangle two macroscopic mirrors (w.r.t. vibrational degrees of freedom)?

Back of the Envelope

$$N \hbar \frac{2\pi}{\lambda_{ph}} = \sqrt{2 m \hbar \omega_m} \Rightarrow N \approx \sqrt{\frac{m \omega_m \lambda_{ph}}{\hbar^2}}$$

$$= \sqrt{\frac{10^{-8}\, 10^{6}\, 10^{-6}}{(10^{-34})^2}}$$

$$= 10^{30}.$$

$$|\sqrt{N} e^{i\varphi}\rangle_{ph} \otimes |0\rangle_m$$

photon pressure

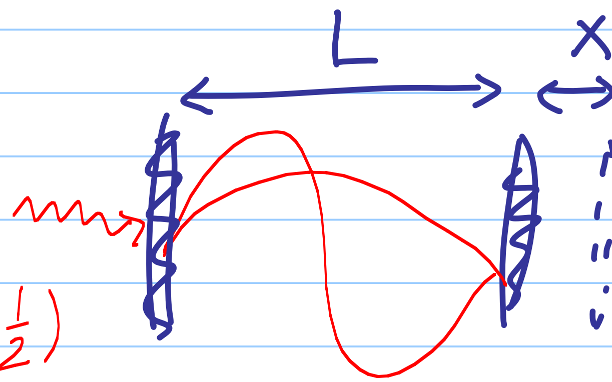
$$\sum_n c_n(\varphi)\, |n\rangle \otimes |\alpha(n,\varphi)\rangle_m$$

entangled

But this assumes mirror in the ground state, which is difficult!

Pressure Coupling

Deriving:



$E = \hbar \omega_{ph}(n+\frac{1}{2})$

$$\omega = \frac{2\pi c}{L}$$

$$\Delta E = E(L+x) - E(L)$$

$$= \hbar(n+\tfrac{1}{2})\left[\frac{2\pi c}{L+x} - \frac{2\pi c}{L}\right]$$

$$= \hbar(n+\tfrac{1}{2})\,\omega_{ph} \cdot \frac{x}{L}$$

quantize $\Rightarrow \hbar\omega_{ph} a^\dagger a \dfrac{\sqrt{\frac{\hbar}{m\omega_m}}}{L}(b+b^\dagger)$

Pace, Collett + Walls, PRA (1993).

A More Realistic Model

$$H = \hbar\omega_{ph} a^\dagger a + \hbar\omega_m b^\dagger b - \hbar g\, a^\dagger a (b^\dagger + b)$$

Start in $|\alpha\rangle\langle\alpha|_{ph} \otimes \rho_m^{th}$

$$\rho_m^{th} = \sum_n \frac{e^{-\beta n \hbar\omega}}{Z} |n\rangle\langle n|$$

$$\beta = \frac{1}{kT}$$

What do we obtain?

Exact Solution

$$U(t) = e^{-i\omega_0 a^\dagger a t} \times e^{i k^2 (a^\dagger a)^2 \Delta(t)} \times D_m\left[\eta(t) k a^\dagger a\right] e^{-i\omega_m b^\dagger b t}$$

$$\Delta(t) = \omega_m t - \sin(\omega_m t)$$

$$\eta(t) = 1 - e^{i\omega_m t}$$

$$k = \frac{g}{\omega_m}$$

$$D_m[\alpha] = e^{\alpha b^\dagger - \alpha^* b}$$

Mancini et al, PRA (1997)

Bose et al, PRA (1997)

Example → General

$$\left(|0\rangle + |1\rangle\right)_{ph} \otimes |0\rangle_m$$

$$\rightarrow |0\rangle|0\rangle + e^{if(t)}|1\rangle|\kappa\eta(t)\rangle$$

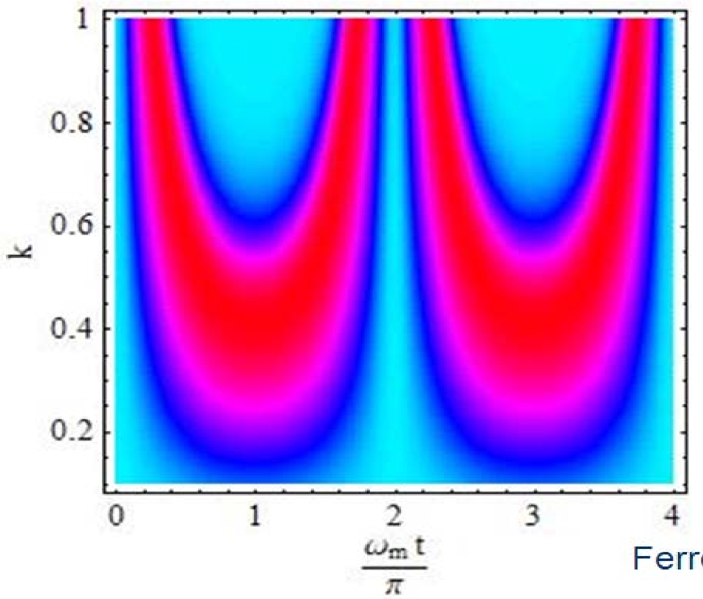
Can do it more generally

$$|\alpha\rangle\langle\alpha|_{ph} \otimes \rho_m^{th}$$

$\rightarrow$ entangled

Ferreira, Guerreiro, Vedral, PRL (2006)

Relevant Plot



Hot Entanglement

w – frequency of mirror
k – strength of interaction

Ferreira, Guerreiro, Vedral, PRL (2006)

We have periodic generation of entanglement depending on k and wm.

Not Surprising

We know that the Jaynes Cummings

$$H = \lambda(\sigma_+ a + \sigma_- a^\dagger)$$

can create entanglement if atom is pure and field is at arbitrarily high T.

*) Has possible implications for Q.C. with single pure qubit.

I. Fuentes, S. Bose, P. Knight, V.V. PRL (2002)

More Serious Model

Include cavity photon loss and the mirror damping.

Resulting evolution is non-unitary and leads to a steady state of "light + mirror".

This state is mixed and we have to decide if it is entangled.

Vitali et al., PRL (2007)

Quantifying Entanglement

Want to know if:

$$\rho_{12} = \sum_i p_i \, \rho_1^i \otimes \rho_2^i$$

Partial transposition eigenvalues provide a witness. $\rho_{12}^{T_2} < 0$

$\Rightarrow \rho_{12}$ entangled.

V.V., Nature (2008)

Gaussian States

Partial transposition is necessary and sufficient for Gaussian states (Simon, 2000).

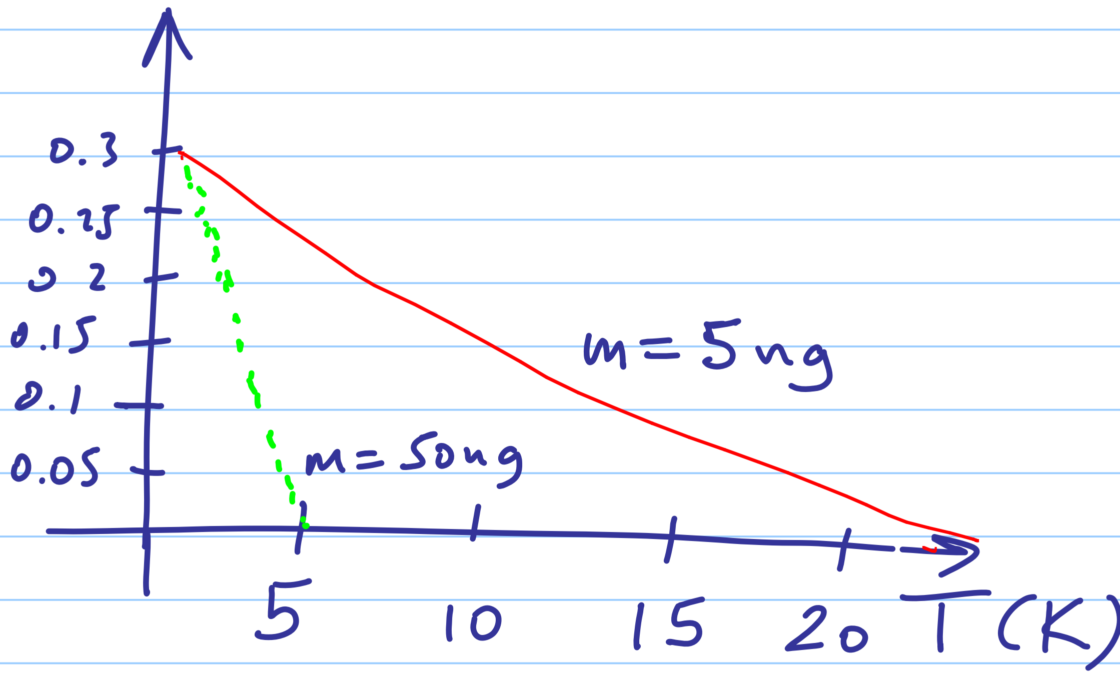
$$\langle (x_1 - x_2)^2 \rangle + \langle (p_1 + p_2)^2 \rangle < 2$$

$$x_1 = a + a^\dagger \qquad p_1 = i(a - a^\dagger)$$

$$x_2 = b + b^\dagger \qquad p_2 = i(b - b^\dagger)$$

Negativity of eigenvalues measures entanglement!

E_N versus T



$L = 1mm$

$P = 50mW$

$\omega_m = 10MHz$

$\gamma_m = 100MHz$

$\mathcal{F} \approx 10^4$

Heuristic Argument

For entangled steady state

1) $\omega_m \ll \gamma_m \ll g\langle a^\dagger a\rangle$

2) $k_B T < \hbar g \langle a^\dagger a\rangle$

$$= \cancel{\hbar}\,\frac{\sqrt{\frac{\hbar}{m\omega_m}}}{\cancel{\kappa}}\,\omega_m \cdot \overbrace{\left(\frac{P}{\cancel{\hbar\omega_m}}\cdot\frac{\cancel{\kappa}}{c}\cdot F\right)}^{\langle a^\dagger a\rangle}$$

$$= \sqrt{\frac{\hbar}{m\omega_m}} \times \frac{P}{c} \times F$$

Putting in $\omega = 10^{-12}\,\mathrm{kg}$ $P = 1\,\mathrm{mW}$ $F = 10^4$

$\Rightarrow$ $\underline{T < 1\,\mathrm{K}}$

One Step Further

$$H_I = \hbar g \, a^\dagger a (b + b^\dagger) + a^\dagger a (c + c^\dagger)$$

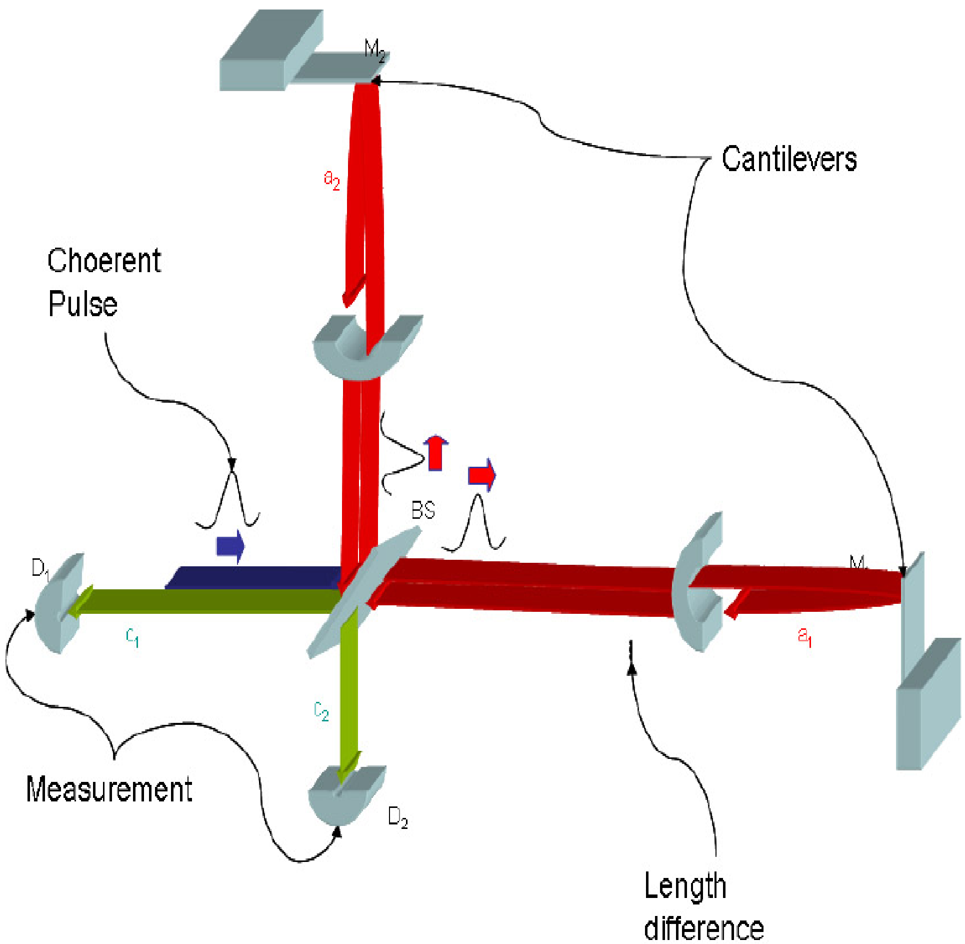
$$(|0\rangle + |1\rangle)_a |0\rangle_b |0\rangle_c$$

$$\rightarrow |0\rangle|0\rangle|0\rangle + |1\rangle|\alpha\rangle|\alpha\rangle$$

trace out $a \Rightarrow |00\rangle\langle 00| + |\alpha\rangle|\alpha\rangle\langle\alpha|\langle\alpha|$

$\Rightarrow$ separable!

Entanglement Swapping



Vacanti, Palma, Paternostro, Vedral,
N. J. Phys (2008).

Maths

$$|1\rangle|0\rangle_{m_1}|0\rangle_{m_2} \rightarrow \left(|01\rangle + |10\rangle\right)|0\rangle_{m_1}|0\rangle_{m_2}$$

→ evolution (BS + photon pressure)

$$|0\rangle|1\rangle|0\rangle_{m_1}|\alpha\rangle_{m_2} + |1\rangle|0\rangle|\alpha\rangle_{m_1}|0\rangle_{m_2}$$

mirrors NOT entangled.

→ evolution (BS)

$$|01\rangle\left(|0\rangle_{m_1}|\alpha\rangle_{m_2} + |\alpha\rangle_{m_1}|0\rangle_{m_2}\right) +$$

$$|10\rangle\left(|0\rangle_{m_1}|\alpha\rangle_{m_2} - |\alpha\rangle_{m_1}|0\rangle_{m_2}\right)$$

measure.

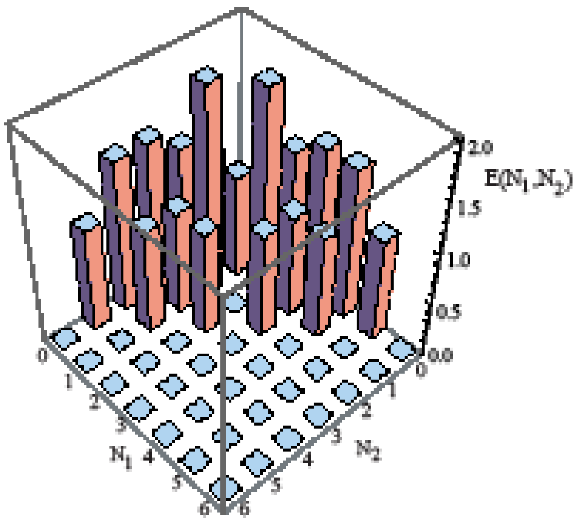
Numbers

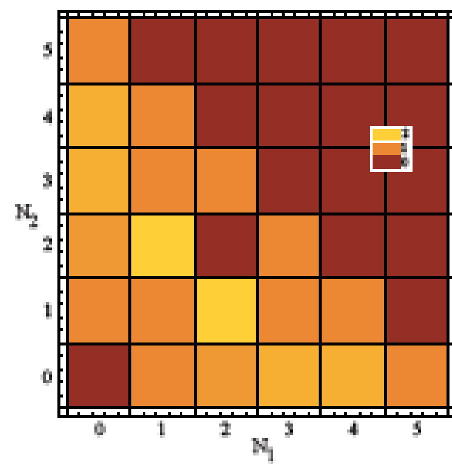
Similar assumptions as before,
but 1) mirrors in ground state
2) $T=0$

Novel : pulse width, timing
at BS.

But, need to check T-finite +
decoherence ... steady state at?

Typical Plot





Entanglement as function of detected photon number.

Would have to average in practice ...

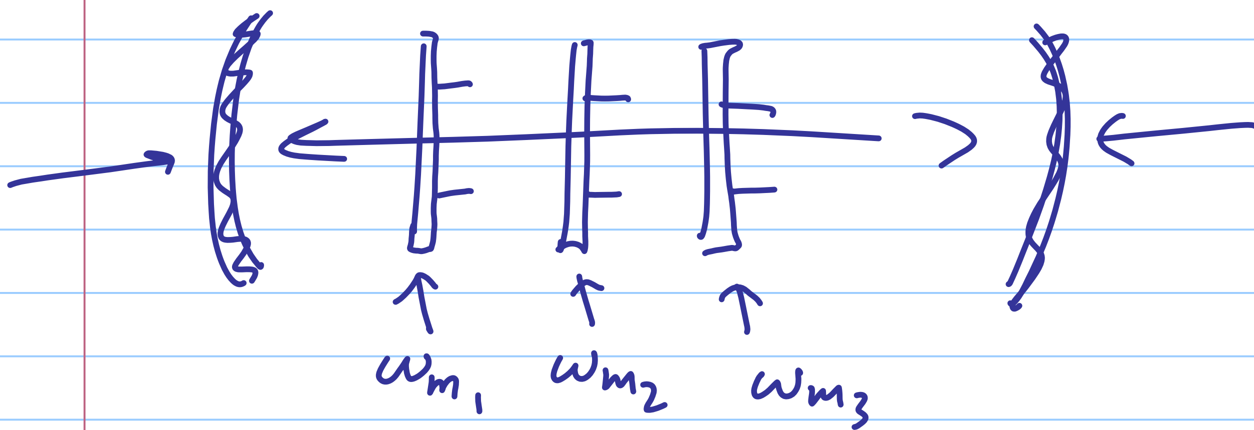
Relevant Experiments

S. Gigan et. al, Nature (2006)

Room T → 10 K (sufficient for entanglement).

Need to get below 1 K, but we are close to it.

More Oscillators?



We know that entanglement exists when

$$k_B T < g\langle a^\dagger a\rangle$$

J. Anders, PRA (2008).

Summary

1) Entangling macroscopic oscillators is possible with present technology

2) Entanglement T_c scales as $\frac{1}{\sqrt{m}} \times \mathcal{F}$ for "reasonable" steady states.

3) Need to think about measuring collective witnesses like

$$\sum_i \left(a_i^\dagger a_{i+1} + h.c\right) ?$$